5.3 – Rational Expressions

Daily Objectives:

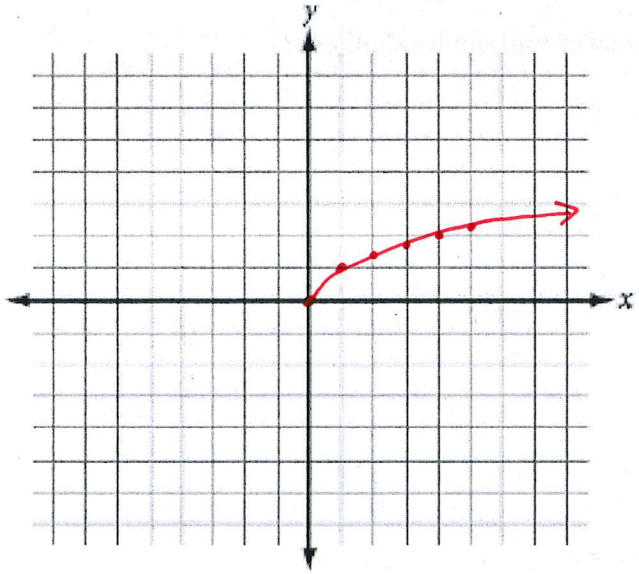
1. Discover that $x^{1/2}$ is equivalent to $\sqrt{x}$.
2. Introduce the root notation $\sqrt[n]{x}$.
3. Define rational exponents as equivalent to roots or roots raised to powers.
4. Formally define the point-ratio form of an exponential function, $y = y_1 b^{x-x_1}$.
5. Use the point-ratio form to find an exponential equation through two points.

Investigation: Getting to the Root ~ Explore the relationship between x and $x^{1/2}$.

Step 1: Use your calculator to create a table for $y = x^{1/2}$ at integer values of x:

x	$y = x^{1/2}$
0	0
1	1
2	1.414
3	1.732
4	2
5	2.236

Step 2: Graph $y = x^{1/2}$. Sketch your graph on this grid. Where have you seen this graph before?



$y = \sqrt{x}$

Step 3: State what you have discovered about raising a number to a power of ½. Include an example with your statement.

RAISING A NUMBER TO THE 1/2 POWER IS THE SAME AS TAKING THE SQUARE ROOT

Step 4: Make a table for $y = 25^x$ with x incrementing by ½. Record this data.

x	$y = 25^x$
½	5
$\frac{2}{2}$	25
$\frac{3}{2}$	125
$\frac{4}{2}$	625
$\frac{5}{2}$	3125
$\frac{6}{2}$	15,625

Step 5: Study your table and explain any relationships you see.

$25^{\frac{1}{2}} = \sqrt{25} = 5$ $\quad 25^{\frac{3}{2}} = (25^{\frac{1}{2}})^3 = 5^3 = 125$

How could you find the value of $49^{\frac{3}{2}}$? Check your answer without a calculator.

$(\sqrt{49})^3 = 7^3 \Rightarrow 7^3 = 343$

Step 6: How could you find the value of $27^{2/3}$ without a calculator?

$(\sqrt[3]{27})^2 = 3^2 = 9$

Test your strategy on $8^{5/3}$.

$(\sqrt[3]{8})^5 = 2^5 = 32$

Definition of Rational Exponents

The power of a power property shows that $a^{m/n} = (a^{1/n})^m$ and $a^{m/n} = (a^m)^{1/n}$, so

$$a^{m/n} = (\sqrt[n]{a})^m \text{ or } \sqrt[n]{a^m} \text{ for } a > 0$$

Example 1: Rewrite the following rational exponent expressions with radicals.

a. $x^{5/3}$

$\sqrt[3]{x^5}$

b. $w^{7/9}$

$(\sqrt[9]{w})^7$

c. $y^{-1/2}$

$\frac{1}{y^{\frac{1}{2}}} = \frac{1}{\sqrt{y}}$

Example 2: Rewrite with rational exponents, and find the positive solution.

a. $\sqrt[4]{a} = 14$

$a^{\frac{1}{4}} = 14$

$(a^{\frac{1}{4}})^4 = 14^4$

$a = 38,416$

b. $\sqrt[9]{b^5} = 26$

$b^{\frac{5}{9}} = 26$

$(b^{\frac{5}{9}})^{\frac{9}{5}} = 26^{\frac{9}{5}}$

$b \approx 352.332$

c. $(\sqrt[3]{c})^8 = 47$

$c^{\frac{8}{3}} = 47$

$(c^{\frac{8}{3}})^{\frac{3}{8}} = 47^{\frac{3}{8}}$

$c \approx 4.237$

d. $3\sqrt[4]{g^5} = 102$

$\frac{3g^{\frac{5}{4}}}{3} = \frac{102}{3}$

$g^{\frac{5}{4}} = 34$

$(g^{\frac{5}{4}})^{\frac{4}{5}} = 34^{\frac{4}{5}}$

$g \approx 16.795$

e. $2x^{6/11} - 19 = 41$

$+19 \quad +19$

$\frac{2x^{\frac{6}{11}}}{2} = \frac{60}{2}$

$x^{\frac{6}{11}} = 30$

$(x^{\frac{6}{11}})^{\frac{11}{6}} = 30^{\frac{11}{6}}$

$x \approx 510.570$

f. $5(\sqrt{c})^7 + 12 = 32$

$5c^{\frac{7}{2}} + 12 = 32$

$-12 \quad -12$

$\frac{5c^{\frac{7}{2}}}{5} = \frac{20}{5}$

$c^{\frac{7}{2}} = 4$

$(c^{\frac{7}{2}})^{\frac{2}{7}} = 4^{\frac{2}{7}}$

$c \approx 1.486$

Point-Ratio Form

If an exponential curve passes through the point (x_1, y_1) and the function values have ratio b for values of x that differ by 1, the point-ratio form of the equation is

$$y = y_1 \cdot b^{x-x_1}$$

Example 3: Casey hit the bell in the school clock tower. Her pressure reader, held nearby, measured the sound intensity (loudness) at 40 lb/in^2 after 4 seconds had elapsed and at 4.7 lb/in^2 after 7.2 seconds had elapsed. She remembers from her science class that sounds decays exponentially.

a. Name two points that the exponential curve must pass through.

(4, 40) (7.2, 4.7)

b. Find an exponential equation that models these data.

$y = 40 \cdot b^{x-4}$ $y = 4.7 \cdot b^{x-7.2}$

$\frac{40 \cdot b^{x-4}}{40} = \frac{4.7b^{x-7.2}}{40}$

$b^{x-4} = \frac{4.7b^{x-7.2}}{40}$

$\frac{b^{x-4}}{b^{x-7.2}} = \frac{4.7}{40}$

$b^{x-4-(x-7.2)} = \frac{4.7}{40}$

$b^{x-4-x+7.2} = \frac{4.7}{40}$

$b^{3.2} = \frac{4.7}{40}$

$b = \left(\frac{4.7}{40}\right)^{\frac{1}{3.2}}$

$b \approx .5121378$

– or –

$y = ab^x$ $y = ab^x$

$\frac{y = ab^x}{y = ab^x}$

$\frac{4.7 = ab^{7.2}}{40 = ab^4}$

$\frac{4.7}{40} = b^{3.2}$

$\left(\frac{4.7}{40}\right)^{\frac{1}{3.2}} = b$

$.5121 \approx b$

$40 = a(.5121378)^4$

$\frac{}{.5121378^4 \quad .5121378^4}$

$581.450 = a$

$y = 581.450(.512)^x$

c. How loud was the bell when it struck at 0 seconds.

$y = 581.450\ (.512)^0$

$y = 581.450$